Inter (Part-I) 2017

Mathematics	Group-II	PAPER: I
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: (16)

(i) **Prove that $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$; justify each step.**

Ans Given, $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$

$$(ab) \cdot \frac{1}{a} \cdot \frac{1}{b} = \left(a \cdot \frac{1}{a}\right)\left(b \cdot \frac{1}{b}\right)$$
$$= 1 \cdot 1$$
$$= 1$$

Thus ab and $\frac{1}{a} \cdot \frac{1}{b}$ are the multiplicative inverse of each other. But multiplicative inverse of ab is $\frac{1}{ab}$.

$$\therefore \quad \frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$$

(ii) **Factorize : $3x^2 + 3y^2$.**

Ans $3x^2 + 3y^2 = 3x^2 - 3(-1)y^2$

$$= 3x^2 - 3i^2y^2$$
$$= (\sqrt{3}\,x)^2 - (\sqrt{3}\,iy)^2$$
$$= (\sqrt{3}\,x + \sqrt{3}\,iy)(\sqrt{3}\,x - \sqrt{3}\,iy)$$

(iii) **Simplify: $(3 - \sqrt{-4})^{-3}$.**

Ans $(3 - \sqrt{-4})^{-3} = (3 - 2i)^{-3}$

$$= \frac{1}{(3 - 2i)^3}$$
$$= \frac{1}{27 - 3.9.2i + 3.3.4(-1) - 8i(-1)}$$
$$= \frac{1}{27 - 54i - 36 + 8i}$$

$$= \frac{1}{-9-46i}$$

$$= \frac{1}{-9-46i} \times \frac{-9+46i}{-9+46i}$$

$$= \frac{-9+46i}{(-9)^2-(46i)^2}$$

$$= \frac{-9+46i}{81-(2116)(-1)}$$

$$= \frac{-9+46i}{81+2116}$$

$$= \frac{-9+46i}{2197}$$

$$\left(3-\sqrt{-4}\right)^{-3} = \frac{-9}{2197} + \frac{46}{2197}i$$

(iv) Write power set of {9, 11} .

Ans Let $A = \{9, 11\}$

$P(A) = \{\phi, \{9\}, \{11\}, \{9, 11\}\}$

(v) Define implication or conditional.

Ans A compound statement of the form if p then q, also written p implies q, is called a conditional or an implication.

(vi) Write the inverse of {(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)}

Ans Relation: {(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)}

Inverse relation: {(3, 1), (5, 2), (7, 3), (9, 4), (11, 5)}

Clearly the relation and its inverse are functions as no one element in a pair is repeated.

(vii) If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, then find A^2.

Ans

$$A^2 = A.A$$

$$= \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$$

$$= \begin{bmatrix} i(i)+0(1) & i(0)+0(-i) \\ 1(i)+(-i)(1) & 1(0)+(-i)(-i) \end{bmatrix}$$

$$= \begin{bmatrix} i^2+0 & 0+0 \\ i-i & 0+i \end{bmatrix}$$

$$= \begin{bmatrix} i^2 & 0 \\ 0 & .i \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix}$$

(viii) Find inverse of $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$.

Ans Let

$$A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 6 & 3 \end{vmatrix}$$

$$= 2(3) - 6(1)$$

$$= 6 - 6$$

$$= 0$$

The further solution does not exist.

(ix) If $B = \begin{bmatrix} 5 & -2 & 5 \\ 3 & -1 & 4 \\ -2 & 1 & -2 \end{bmatrix}$, then find B_{21}, B_{22}.

Ans Given,

$$B = \begin{bmatrix} 5 & -2 & 5 \\ 3 & -1 & +4 \\ -2 & 1 & -2 \end{bmatrix}$$

$$B_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 5 \\ 1 & -2 \end{vmatrix}$$

$$= (-1)^3 [(-2)(-2) - 1(5)]$$

$$= -1 (4 - 5)$$

$$= 1$$

$$B_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 5 \\ -2 & -2 \end{vmatrix}$$

$$= (-1)^2 [5(-2) - (-2)(5)]$$

$$= 1(-10 + 10)$$

$$= 0$$

(x) Solve $x^2 - 7x + 10 = 0$ by factorization.

Ans

$$x^2 - 7x + 10 = 0$$

$$x^2 - 2x - 5x + 10 = 0$$

$$x(x - 2) - 5(x - 2) = 0$$

$$(x - 2)(x - 5) = 0$$

$$x - 2 = 0 \quad ; \quad x - 5 = 0$$

$$\boxed{x = 2} \quad ; \quad \boxed{x = 5}$$

Thus, solution set is {2, 5}.

(xi) **Prove that $1 + \omega + \omega^2 = 0$.**

Ans We know that cube roots of unity are:

$$1, \frac{-1+\sqrt{3}\,i}{2} \text{ and } \frac{-1-\sqrt{3}\,i}{2}$$

If $\omega = \frac{-1+\sqrt{3}\,i}{2}$,

then $\omega^2 = \frac{-1-\sqrt{3}\,i}{2}$

Sum of all the three cube roots

$$1 + \omega + \omega^2 = 1 + \frac{-1+\sqrt{3}\,i}{2} + \frac{-1-\sqrt{3}\,i}{2}$$

$$= \frac{2-1+\sqrt{3}\,i-1-\sqrt{3}\,i}{2}$$

$$= \frac{0}{2} = 0$$

Hence sum of cube roots of unity

$1 + \omega + \omega^2 = 0$.

(xii) **The sum of a positive number and its reciprocal is $\frac{26}{5}$. Find the number.**

Ans Let the number = x

Condition given : $x + \frac{1}{x} = \frac{26}{5}$

$$\Rightarrow \quad 5x^2 - 26x + 5 = 0$$

$$5x^2 - 25x - x + 5 = 0$$

$$5x(x-5) - 1(x-5) = 0$$

$$(x-5)(5x-1) = 0$$

$x - 5 = 0 \quad ; \quad 5x - 1 = 0$

$x = 5 \quad ; \quad 5x = 1$

$\quad ; \quad x = \frac{1}{5}$

$\therefore$ Required number $= 5, \frac{1}{5}$.

3. Write short answers to any EIGHT (8) questions: (16)

(i) **Define proper rational fraction.**

Ans A rational fraction $\frac{P(x)}{Q(x)}$ is called a proper rational fraction, if the degree of the polynomial P(x) in the numerator is less than the degree of the polynomial Q(x) in the denominator. For example, $\frac{3}{x+1}, \frac{2x-5}{x^2+4}$ and $\frac{9x^2}{x^3-1}$ are proper rational fractions or proper fractions.

(ii) Which term of the arithmetic sequence 5, 2, –1, ---- is – 85?

Ans Given, AP , 5, 2, –1, - - - , –85

Here $a = 5,$

$d = 2 - 5 = -3$

$a_n = -85$

$n = ?$

$a_n = a + (n-1)d$

$-85 = 5 + (n-1)(-3)$

$-85 = 5 - 3n + 3$

$-85 = 8 - 3n$

$3n = 8 + 85$

$3n = 93$

$n = \frac{93}{3}$

$n = 31$

Thus $a_{31} = -85$

(iii) Find the next two terms of sequence –1, 2, 12, 40, – .

Ans By inspection, the terms can be written as

$-1 \times 1, 1 \times 2, 3 \times 4, 5 \times 8$ (A)

The sequence of first term is –1, 1, 3, 5, — giving common difference as 2, therefore, the next two terms are 7 and 9.

The sequence of second term is 1, 2, 4, 8, —, — giving common ratio as 2, therefore, the next two terms are 16 and 32.

Hence the next two terms of the given sequence are:

$7 \times 16, 9 \times 32$, *i.e.*, 112 and 288.

(iv) Show that the reciprocals of terms of geometric sequence a_1, a_1r^2, a_1r^4, - - - - form another geometric sequence.

Ans The given geometric sequence are:

$a_1, a_1r^2, a_1r^4, ----$

The reciprocals of above sequence are

$\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a_1r^4}, ----$

Here

$$r = \frac{\left(\frac{1}{a_1r^2}\right)}{\left(\frac{1}{a_1}\right)}$$

$$= \frac{1}{a_1r^2} \cdot a_1$$

$$= \frac{1}{r^2}$$

also

$$r = \frac{\frac{1}{a_1r^4}}{\left(\frac{1}{a_1r^2}\right)}$$

$$= \frac{1}{a_1r^4} \times a_1r^2$$

$$= \frac{1}{r^2}$$

As the common ratio is same, so the sequence $\frac{1}{a_1}, \frac{1}{a_1r^2}, \frac{1}{a_1r^4}, ----$ is G.P.

(v) First term of harmonic sequence is $-\frac{1}{3}$ and fifth term is $\frac{1}{5}$. Find 9^{th} term.

Ans In harmonic progression (H.P)

$a_1 = \frac{-1}{3}$, $a_5 = \frac{1}{5}$

In arithmetic progression (A.P):

$a_1 = -3, a_5 = 5$

As $a = a_1$

Now in A.P

$$a_1 = -3 \quad \text{(i)}$$

$$a_5 = 5$$

$$a + 4d = 5 \quad \text{(ii)}$$

By putting equation (i) in equation (ii), we get

$$-3 + 4d = 5$$

$$4d = 5 + 3$$

$$4d = 8$$

$$d = 2$$

Thus, $a = -3, d = 2$

Now, $a_9 = a + (9 - 1)d$

$$= -3 + 8(2)$$

$$= -3 + 16$$

$$= 13$$

Thus $a_9 = \dfrac{1}{13}$ in harmonic sequence.

(vi) Find the value of n when $^{11}P_n = 11 . 10 . 9$.

Ans $^{11}P_n = \dfrac{11 . 10 . 9 . 8!}{8!}$

$$\frac{11!}{(11-n)!} = \frac{11!}{8!}$$

$$11 - n = 8 \qquad n = 3$$

(vii) Find the value of n and r when $^nC_r = 35$ and $^nP_r = 210$.

Ans $^nC_r = 35$

$$\frac{n!}{(n-r)!\,r!} = 35 \quad (1)$$

$^nP_r = 210$

$$\frac{n!}{(n-r)!} = 210 \quad (2)$$

Using eq. (2) in eq. (1),

$$\frac{210}{r!} = 35$$

$$\Rightarrow \quad r! = \frac{210}{35}$$

$$r! = 6$$

$$r! = 3!$$

$$\boxed{r = 3}$$

Put in (2),

$$\frac{n!}{(n-3)!} = 210$$

$$\frac{n!}{(n-3)!} = \frac{2 \times 3 \times 7 \times 5 \times 1 \times 4 \times 6}{1 \times 4 \times 6}$$

$$\frac{n!}{(n-3)!} = \frac{7!}{4!}$$

$$\frac{n!}{(n-3)!} = \frac{7!}{(7-3)!}$$

$${}^nP_3 = {}^7P_3$$

$$\boxed{n = 7}$$

(viii) A coin is tossed four times. Find probability of the events happening 2 heads and 2 tails.

Ans As the coin is tossed four times, thus the total possible outcomes are:

$$2^4 = 16$$

Here: heads = H, tails = T

Favourable outcomes are 6, *i.e.*,

HHTT, THHT, HTHT, HTTH, THTH, TTHH

$$\therefore \quad P(\text{2 heads and 2 tails}) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{6}{16}$$

$$= \frac{3}{8}$$

(ix) A die is thrown. Find probability that the dots on the top are prime numbers or odd numbers.

Ans Here $S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

Let A = Set of prime numbers

$= \{2, 3, 5\}$

$n(A) = 3$

Let B = Set of odd numbers

$= \{1, 3, 5\}$

$n(B) = 3$

$\therefore \quad A \cap B = \{2, 3, 5\} \cap \{1, 3, 5\}$

$= \{3, 5\}$

$\Rightarrow \quad n(A \cap B) = 2$

Now

$$P(A) = \frac{n(A)}{n(S)} \frac{3}{6} = \frac{1}{2}$$

$$P(B) = \frac{n(B)}{n(S)} \frac{3}{6} = \frac{1}{2}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} \frac{2}{6} = \frac{1}{3}$$

Since A and B are overlapping sets.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{3}$$

$$= \frac{2}{3}$$

(x) Use mathematical induction to prove that $1 + \frac{1}{2} + \frac{1}{4} + --- + \frac{1}{2^{n-1}} = 2\left[1 - \frac{1}{2^n}\right]$ for n = 1, 2.

Ans **Condition 1:**

For n = 1,

$$\frac{1}{2^{1-1}} = 2\left[1 - \frac{1}{2^1}\right]$$

$$\frac{1}{2^0} = 2\left(\frac{2-1}{2}\right)$$

$$1 = 1$$

So condition 1 is satisfied.

Condition 2:

Suppose the formula is true for n = k, *i.e.*,

$$1 + \frac{1}{2} + \frac{1}{4} + --- + \frac{1}{2^{k-1}} = 2\left[1 - \frac{1}{2^k}\right]$$

Condition 3:

Add 2^k on both sides of the above equation,

$$1 + \frac{1}{2} + \frac{1}{4} + --- + \frac{1}{2^{k-1}} + \frac{1}{2^k} = 2\left[1 - \frac{1}{2^k}\right] + \frac{1}{2^k}$$

$$1 + \frac{1}{2} + \frac{1}{4} + --- + \frac{1}{2^{k-1}} + \frac{1}{2^k} = 2 - 2 \cdot \frac{1}{2^k} + \frac{1}{2^k}$$

$$1 + \frac{1}{2} + \frac{1}{4} + -- + \frac{1}{2^{k-1}} + \frac{1}{2^k} = 2 - \frac{1}{2^k}$$

$$1+\frac{1}{2}+\frac{1}{4}+---+\frac{1}{2^{k-1}}+\frac{1}{2^k}=2\left[1-\frac{1}{2\times 2^k}\right]$$

$$1+\frac{1}{2}+\frac{1}{4}+---+\frac{1}{2^{k-1}}+\frac{1}{2^k}=\left[1-\frac{1}{2^{k+1}}\right]$$

Hence by the principle of mathematical induction the formula is true for all natural numbers n.

(xi) **Show that $\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\ldots..+n\binom{n}{n}=n\,2^{n-1}$.**

Ans $\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\ldots..+n\binom{n}{n}=n+\frac{2n(n-1)}{2!}+\frac{3n(n-1)(n-2)}{3!}$

$$+\ldots.+n.1$$

$$=n.\left[1+(n-1)+\frac{(n-1)(n-2)}{2!}+\ldots.+1\right]$$

$$=n.\left[\binom{n-1}{0}+\binom{n-1}{1}+\binom{n-1}{2}+\ldots..+\binom{n-1}{n-1}\right]$$

$$=n.2^{n-1}$$

(xii) **Expand $(1-2x)^{1/3}$ up to three terms.**

Ans $(1-2x)^{1/3}=1+\frac{1}{3}(-2x)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}(-2x)^2+\ldots\ldots$

$$=1-\frac{2}{3}x+\frac{\frac{1}{3}\left(\frac{-2}{3}\right)}{2.1}(4x^2)+\ldots\ldots$$

$$=1-\frac{2}{3}x-\frac{4}{9}x^2+\ldots\ldots$$

Putting x = 0.1 in the above equation we have

$$(1-2(0.1))^{1/3}=1-\frac{2}{3}(0.1)-\frac{4}{9}(0.1)^2\ldots..$$

$$(1-0.2)^{1/3}=1-\frac{0.2}{3}-\frac{0.04}{9}\ldots..$$

$$(0.8)^{1/3}\approx 1-0.6666-0.00444$$

$$(0.8)^{1/3}\approx 0.9289$$

4. Write short answers to any NINE (9) questions: (18)

(i) **Find l, when $\theta = 65^\circ 20'$, r = 18 mm.**

Ans Given, r = 18 mm

$$\pi=\frac{22}{7}$$

$$\theta = 65^\circ 20'$$
$$= \left(65 + \frac{20}{60}\right)^\circ$$
$$= \left(65 + \frac{1}{3}\right)^\circ$$
$$= \frac{196^\circ}{3}$$
$$\theta = \frac{196}{3} \times \frac{\pi}{180} \text{ radians}$$
$$= 1.1403 \text{ radians}$$

As

$$l = r\theta$$
$$l = 18(1.1403)$$
$$l = 20.53 \text{ mm}$$

(ii) If $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$, then find x.

Ans $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$

$$(1) - \left(\frac{1}{2}\right)^2 = x\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(\sqrt{3})$$
$$1 - \frac{1}{4} = x\frac{\sqrt{3}}{2}$$
$$\frac{3}{4} = \frac{x\sqrt{3}}{2}$$
$$\frac{3}{4} \times \frac{2}{\sqrt{3}} = x$$
$$\frac{\sqrt{3}}{2} = x$$
$$\Rightarrow \quad x = \frac{\sqrt{3}}{2}$$

(iii) Prove that $\frac{\sin\theta}{1 + \cos\theta} + \cot\theta = \operatorname{cosec}\theta$.

Ans

$$\text{L.H.S} = \frac{\sin\theta}{1 + \cos\theta} + \cot\theta$$
$$= \frac{\sin\theta}{1 + \cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos\theta\,(1+\cos\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{\sin^2\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)(\sin\theta)}$$

As $\quad \sin^2\theta + \cos^2\theta = 1$

So, $\quad \text{L.H.S} = \frac{1+\cos\theta}{(1+\cos\theta)\,(\sin\theta)}$

$$= \frac{1}{\sin\theta}$$

$$= \operatorname{cosec}\theta$$

$$= \text{R.H.S}$$

(iv) If α, β, γ are the angles of a ΔABC, then prove that $\cos\left(\frac{\alpha+\beta}{2}\right) = \sin\frac{\gamma}{2}$.

Ans Since α, β and γ are the angles of a triangle

$\therefore \quad \alpha + \beta + \gamma = 180^\circ$

$$\alpha + \beta = 180^\circ - \gamma$$

$$\frac{\alpha+\beta}{2} = \frac{180^\circ - \gamma}{2}$$

$$\frac{\alpha+\beta}{2} = \frac{180^\circ}{2} - \frac{\gamma}{2}$$

$$\frac{\alpha+\beta}{2} = 90^\circ - \frac{\gamma}{2}$$

$$\cos\left(\frac{\alpha+\beta}{2}\right) = \cos\left(90^\circ - \frac{\gamma}{2}\right)$$

$$= \sin\frac{\gamma}{2}$$

(v) Prove that $\sin(180^0 + \alpha)\sin(90^\circ - \alpha) = -\sin\alpha\cos\alpha$.

Ans $\sin(180^\circ + \alpha)\sin(90^\circ - \alpha) = -\sin\alpha\cos\alpha \qquad$ (i)

$\sin(180^\circ + \alpha) = \sin 180^\circ \cos\alpha + \cos 180^\circ \sin\alpha$

$= 0 \,.\, \cos\alpha + (-1)\sin\alpha$

$= 0 - \sin\alpha$

$\sin(180^\circ + \alpha) = -\sin\alpha \qquad$ (ii)

$\sin(90^\circ - \alpha) = \sin 90^\circ \cos\alpha - \cos 90^\circ \sin\alpha$

$= 1 \,.\, \cos\alpha - 0 \,.\, \sin\alpha$

$= \cos\alpha - 0$

$\sin(90^\circ - \alpha) = \cos\alpha$ (iii)

By combining (ii) and (iii), we get (i), *i.e.*, proved

$$\sin(180^\circ + \alpha)\sin(90^\circ - \alpha) = -\sin\alpha\cos\alpha$$

(vi) Prove that $\sin(45^\circ + \alpha) = \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$.

Ans L.H.S $= \sin(45^\circ + \alpha)$

$$= \sin 45^\circ \cos\alpha + \cos 45^\circ \sin\alpha$$

$$= \frac{1}{\sqrt{2}}\cos\alpha + \frac{1}{\sqrt{2}}\sin\alpha$$

$$= \frac{1}{\sqrt{2}}(\sin\alpha + \cos\alpha)$$

$$= \text{R.H.S}$$

(vii) Find the period of $\cot\frac{x}{2}$.

Ans $\because \cot\frac{x}{2} = \cot\left(\frac{x}{2} + \pi\right)$

$$= \cot\frac{1}{2}(x + 2\pi)$$

Hence period of $\cot\frac{x}{2}$ is 2π.

(viii) At the top of a cliff 80 m high, the angle of depression of a boat is 12°. How far is the boat from the cliff?

Ans From right ΔABC,

$$\tan 12^\circ = \frac{80}{x}$$

and

$$x = \frac{80}{\tan 12^\circ}$$

$$x = 376.3 \text{ m}$$

(ix) Solve the ΔABC in which $a = 3$, $c = 6$ and $\beta = 36^\circ 20'$.

Ans $a = 3,\ c = 6,\ \beta = 36^\circ 20'$

$\alpha + \gamma = 180^\circ - \beta \Rightarrow \alpha + \gamma = 143^\circ 40'$ (i)

By Law of tangent

$$\frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{\gamma+\alpha}{2}\right)} = \frac{c-a}{c+a} \Rightarrow \frac{\tan\left(\frac{\gamma-\alpha}{2}\right)}{\tan\left(\frac{143^\circ\,40'}{2}\right)} = \frac{6-3}{6+3}$$

$$\Rightarrow \quad \tan\left(\frac{\gamma-\alpha}{2}\right) = \frac{1}{3}(3.05) = 1.02$$

$$\Rightarrow \quad \left(\frac{\gamma-\alpha}{2}\right) = \tan^{-1}(1.02) = 45.45^\circ$$

$$\Rightarrow \quad \gamma - \alpha = 90.90^\circ = 90^\circ\,36' \qquad \text{(ii)}$$

Adding (i) and (ii), we get

$$\Rightarrow \quad \gamma = 117^\circ\,17'$$

Put in (i), we get

$$\alpha = 26^\circ\,23'$$

By Law of sines,

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$$

$$\Rightarrow \quad b = \frac{a\sin\beta}{\sin\alpha} = \frac{3(0.59)}{0.44} = 3.998$$

Hence $\alpha = 26^\circ\,23'$, $\gamma = 117^\circ\,17'$, $b = 3.998$

(x) Find the smallest angle of the ΔABC, when $a = 37.34$, $b = 3.24$, $c = 35.06$

Ans By cosine formula

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$= \frac{(37.34)^2 + (35.06)^2 - (3.24)^2}{2(37.34)(35.06)}$$

$$= \frac{2612.98}{2618.28} = 0.998$$

$$\beta = \cos^{-1}(0.998) = 3^\circ\,37'\,28''$$

(xi) Without using calculator, show that $\cos^{-1}\frac{4}{5} = \cot^{-1}\frac{4}{3}$

Ans Let, $\cos^{-1}\frac{4}{5} = \alpha$ (i)

$$\frac{4}{5} = \cos\alpha$$

Now $\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha}$

$$= \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$$

$$= \frac{\frac{4}{5}}{\sqrt{1-\left(\frac{4}{5}\right)^2}}$$

$$= \frac{\frac{4}{5}}{\sqrt{\frac{9}{25}}}$$

$$= \frac{\frac{4}{5}}{\frac{3}{5}}$$

$$= \frac{4}{5} \times \frac{5}{3}$$

$$\cot\alpha = \frac{4}{3}$$

$$\alpha = \cot^{-1}\frac{4}{3} \qquad \text{(ii)}$$

By (i) and (ii),

$$\cos^{-1}\frac{4}{5} = \cot^{-1}\frac{4}{3} \text{ proved}$$

(xii) Solve the equation tan x = –1 in [0, 2π].

Ans $\tan x = -1$

tan x is negative in second and fourth quadrants with reference angle $x = \frac{\pi}{4}$.

$\therefore \quad x = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad$ where $\quad x \in [0, 2\pi]$

As π is the period of tan x,

$\therefore \quad$ General value of x is $\frac{3\pi}{4} + n\pi, \quad n \in Z$

$\therefore \quad$ Solution set $= \left\{\frac{3\pi}{4} + n\pi\right\}, n \in Z$

(xiii) Find solution of the equation $\sec\theta = -\frac{2}{\sqrt{3}}$ in $[0, 2\pi]$.

Ans

$$\sec\theta = \frac{-2}{\sqrt{3}}$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$\therefore$ $\cos\theta$ is negative in second and third quadrants with the angle $\theta = \frac{-\pi}{6}$.

$$\therefore \quad \theta = \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

and $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$$\frac{\pi}{6}, \frac{7\pi}{6}$$

SECTION-II

NOTE: Attempt any THREE (3) questions.

Q.5.(a) For any three sets A, B and C, prove that $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$. (5)

Ans Let $x \in A \cap (B\cup C)$

$\Rightarrow$ $x \in A$ or $x \in B \cup C$

$\Rightarrow$ If $x \in A$, it must belong to $A \cap B$ and $x \in A \cap C$

$x \in (A\cap B) \cup (A\cap C)$

Also if $x \in B \cup C$, then $x \in B$ and $x \in C$

$\Rightarrow$ $x \in A \cap B$ and $x \in A \cap C$

$\Rightarrow$ $x \in (A \cap B) \cap (A \cup C)$

Thus $A\cap(B\cup C) \subseteq (A\cap B) \cup (A\cap C)$ (1)

Conversely, suppose that

$y \in (A \cap B) \cup (A \cap C)$

There are two cases to consider:

$y \in A$, $y \notin A$

In the first case, $y \in A \cap (B \cup C)$

If $y \notin A$, it must belong to B as well as C *i.e.,* $y \in B\cup C$

$\therefore$ $y \in A \cap (B \cup C)$

So in either case,

$$y \in (A \cap B) \cup (A \cap C)$$

$$\Rightarrow \quad y \in A \cap (B \cup C)$$

Thus $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ (2)

From (1) and (2),

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(b) Verify that $(AB)^t = B^tA^t$ if $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix}$. (5)

Ans Here

$$AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1-3+0 & 1-2-2 \\ 0+9+0 & 0+6-1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 \\ 9 & 5 \end{bmatrix}$$

$$\therefore \quad (AB)^t = \begin{bmatrix} -2 & 9 \\ -3 & 5 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}, \quad B^t = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\therefore \quad B^tA^t = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3+0 & 0+9+0 \\ 1-2-2 & 0+6-1 \end{bmatrix} = \begin{bmatrix} -2 & 9 \\ -3 & 5 \end{bmatrix}$$

$$\therefore \quad (AB)^t = \begin{bmatrix} -2 & 9 \\ -3 & 5 \end{bmatrix} = B^tA^t$$

Q.6.(a) Solve $3^{2x-1} - 12.3^x + 81 = 0$. (5)

Ans

$$3^{2x-1} - 12.3^x + 81 = 0$$

$$3^{-1}.3^{2x} - 12.3^x + 81 = 0$$

Put $3^x = y$,

then $\frac{1}{3}.3^{2x} - 12.3^x + 81 = 0$

$$\frac{1}{3}y^2 - 12y + 18 = 0$$

$$y^2 - 36y + 243 = 0$$
$$(y - 27)(y - 9) = 0$$
$$y = 9, 27$$

Now $y = 9$

$$3^x = 9 = (3)^2$$
$$x = 2$$

and $y = 27$

$$3^x = 27 = (3)^3$$
$$x = 3$$

$\therefore$ S.S. = {2, 3}

(b) Resolve into partial fraction $\dfrac{x^2}{(x-2)(x-1)^2}$. **(5)**

Ans Let $\dfrac{x^2}{(x-2)(x-1)^2} = \dfrac{A}{x-2} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$ (1)

Multiply by $(x - 2)(x - 1)^2$ on both sides,

$$x^2 = A(x-1)^2 + B(x-1)(x-2) + C(x-2) \quad (2)$$

Put $x = 1$ in eq. (2) we have

$$1 = C(1 - 2)$$
$$\boxed{C = -1}$$

Put $x = 2$ in eq. (2), we have

$$4 = A(2 - 1)^2$$
$$\boxed{A = 4}$$

Again from eq. (2)

$$x^2 = A(x^2 - 2x + 1) + B(x^2 - 3x + 2) + C(x - 2)$$
$$x^2 + 0.x + 0 = (A + B)x^2 + (-2A - 3B + C)x + (A + 2B - 2C)$$

Compare the coefficients of x^2, we have

$$A + B = 1$$
$$B = 1 - 4$$
$$B = -3$$

Thus $A = 4$, $B = -3$, $C = -1$

Putting the values of A, B and C in eq. (1), we have

$$\frac{x^2}{(x-2)(x-1)^2} = \frac{4}{x-2} - \frac{3}{x-1} - \frac{1}{(x-1)^2}$$

Hence partial fractions are $\dfrac{4}{x-2} - \dfrac{3}{x-1} - \dfrac{1}{(x-1)^2}$

Q.7.(a) If $y = \frac{x}{2} + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \text{------}$ and $0 < x < 2$, then prove that $x = \frac{2y}{1+y}$. (5)

Ans $a = \frac{x}{2}$, $r = \frac{x}{2}$, $s = y$

$$y = \frac{a}{1-r}$$

$$= \frac{x/2}{1-x/2} = \frac{x}{2-x}$$

$$\frac{1}{y} = \frac{2-x}{x} = \frac{2}{x} - 1$$

$$\frac{1}{y} + 1 = \frac{2}{x}$$

$$\frac{1+y}{y} = \frac{2}{x}$$

$$\frac{y}{1+y} = \frac{x}{2}$$

$$\frac{2y}{1+y} = x$$

(b) Use the mathematical induction to prove that $1 + 4 + 7 + \text{-----} + (3n-2) = \frac{n(3n-1)}{2}$. (5)

Ans Let $S(n)$ be the given statement

i.e., $S(n) = 1 + 4 + 7 + \ldots\ldots + (3n-2) = \frac{n(3n-1)}{2}$ (i)

If $n = 1$, we have

$$1 = \frac{1(3-1)}{2} = \frac{3-1}{2} \text{ which is true.}$$

Thus the condition (I) is satisfied as S(I) is true.

Suppose that $S(n)$ is true for

$$n = k$$

i.e., $1 + 4 + 7 \ldots + (3k-2) = \frac{k(3k-1)}{2}$ (A)

Adding $(3k+1)$ B.S

$$1 + 4 + 7 + \ldots + (3k-2) + (3k+1) = \frac{k(3k-1)}{2} + (3k+1)$$

$$= \frac{1}{2}[(3k^2 - k) + 2(3k + 1)] = \frac{1}{2}[(3k^2 + 5k + 2)]$$

$$= \frac{1}{2}[(3k^2 + 3k + 2k + 2)] = \frac{1}{2}[3k(k + 1) + 2(k + 1)]$$

$$= \frac{1}{2}(k + 1)(3k + 2)$$

or $\quad 1 + 4 + 7 + \ldots + (3k + 3 - 2) = \frac{1}{2}(k + 1)(3k + 3 - 1)$

$$1 + 4 + 7 + \ldots + (3k + 3 - 2) = \frac{1}{2}(k + 1)[3(k + 1) - 1] \quad \text{(B)}$$

The equation (B) shows that $S(k + 1)$ follows from $S(k)$. So the condition (2) is satisfied.

Since, both the conditions are satisfied.

$S_{(n)}$ is true for all integers n.

Q.8.(a) Show that $\frac{1}{1 + \sin\theta} + \frac{1}{1 - \sin\theta} = 2\sec^2\theta$. **(5)**

Ans L.H.S $= \frac{1}{1 + \sin\theta} + \frac{1}{1 - \sin\theta}$

$$= \frac{1(1 - \sin\theta) + 1(1 + \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}$$

$$= \frac{1 - \sin\theta + 1 + \sin\theta}{1 - \sin^2\theta}$$

As $\quad \sin^2\theta + \cos^2\theta = 1$

$$\cos^2\theta = 1 - \sin^2\theta$$

So, L.H.S $= \frac{2}{\cos^2\theta} = 2 \cdot \frac{1}{\cos^2\theta}$

$$= 2\sec^2\theta = \text{R.H.S}$$

(b) Prove that $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$. **(5)**

Ans L.H.S $= \frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta}$

$$= \frac{2\sin\frac{\theta + 3\theta}{2}\cos\frac{\theta - 3\theta}{2} + 2\sin\frac{5\theta + 7\theta}{2}\cos\frac{5\theta - 7\theta}{2}}{2\cos\frac{\theta + 3\theta}{2}\cos\frac{\theta - 3\theta}{2} + 2\cos\frac{5\theta + 7\theta}{2}\cos\frac{5\theta - 7\theta}{2}}$$

$$= \frac{2\sin 2\theta \cos\theta + 2\sin 6\theta \cos(-\theta)}{2\cos 2\theta \cos\theta + 2\cos 6\theta \cos(-\theta)} \quad (\because \cos(-\theta) = \cos\theta)$$

$$= \frac{2\cos\theta\,[\sin 2\theta + \sin 6\theta]}{2\cos\theta\,[\cos 2\theta + \cos 6\theta]}$$

$$= \frac{\sin 2\theta + \sin 6\theta}{\cos 2\theta + \cos 6\theta} = \frac{2\sin\frac{2\theta+6\theta}{2}\cos\frac{2\theta-6\theta}{2}}{2\cos\frac{2\theta+6\theta}{2}\cos\frac{2\theta-6\theta}{2}}$$

$$= \frac{\sin 4\theta \cos 2\theta}{\cos 4\theta \cos 2\theta} = \tan 4\theta = \text{R.H.S}$$

Q.9.(a) Show that $r_1 = 4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$. **(5)**

Ans R.H.S $= 4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

$$= 4\cdot\frac{abc}{4\Delta}\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{s(s-b)}{ca}}\sqrt{\frac{s(s-c)}{ab}}$$

$$= \frac{s(s-b)(s-c)}{\Delta} = \frac{s(s-a)(s-b)(s-c)}{\Delta(s-a)}$$

$$= \frac{\Delta^2}{\Delta(s-a)} = \frac{\Delta}{s-a}$$

$$= r_1 = \text{L.H.S}$$

Hence, $r_1 = 4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

(b) Prove that $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$. **(5)**

Ans L.H.S

$$\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \tan^{-1}\left[\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\times\frac{3}{5}}\right] - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left[\frac{\frac{15+12}{20}}{1-\frac{9}{20}}\right] - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left[\frac{\frac{27}{20}}{\frac{20-9}{20}}\right] - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\frac{27}{11} - \tan^{-1}\frac{8}{19}$$

$$= \tan^{-1}\left[\frac{\frac{27}{11} - \frac{8}{19}}{1 + \frac{27}{11}\times\frac{8}{19}}\right]$$

$$= \tan^{-1}\frac{425}{425} = \tan^{-1}(1) = \frac{\pi}{4}$$